

Exact renormalization of the photino mass in softly broken $\mathcal{N} = 1$ SQED with N_f flavors regularized by higher derivatives

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Abstract

We consider the softly broken $\mathcal{N} = 1$ supersymmetric electrodynamics, regularized by higher derivatives. For this theory we demonstrate that the renormalization of the photino mass is determined by integrals of double total derivatives in the momentum space in all orders. Consequently, it is possible to derive the NSVZ-like exact relation between the photino mass anomalous dimension and the anomalous dimension of the matter superfields in the rigid theory by direct summation of supergraphs. It is important that both these renormalization group functions are defined in terms of the bare coupling constant, so that the considered NSVZ-like relation is valid independently of the subtraction scheme in the case of using the higher derivative regularization. The factorization of integrals defining the photino mass renormalization into integrals of double total derivatives is verified by an explicit two-loop calculation.

1 Introduction

An interesting feature of $\mathcal{N} = 1$ supersymmetric gauge theories is the existence of the relation between the β -function and the anomalous dimension of the matter superfields, which is called "the exact NSVZ β -function" [1, 2, 3, 4]. (For the pure Yang–Mills theory it gives the exact β -function in the form of a geometric series.) A similar relation also exists in softly broken $\mathcal{N} = 1$ supersymmetric theories for the anomalous dimension of the gaugino mass [5, 6, 7]. This relation can be presented in the form [5]

$$\frac{\alpha m}{\beta(\alpha)} = \text{RGI}, \quad (1)$$

where $\beta(\alpha)$ in Eq. (1) is the exact NSVZ β -function for the considered theory, m is the gaugino mass, and RGI denotes that this expression is the renormalization group (RG) invariant.

In this paper we will discuss the softly broken $\mathcal{N} = 1$ supersymmetric electrodynamics (SQED) with N_f flavors, for which the exact NSVZ β -function is written as [8, 9]

$$\beta(\alpha) = \frac{\alpha^2 N_f}{\pi} (1 - \gamma(\alpha)) \quad (2)$$

and relates the β -function to the anomalous dimension of the matter superfields $\gamma(\alpha)$. Differentiating Eq. (1) with respect to $\ln \mu$, where μ is the renormalization point, after some transformations one obtains

$$\frac{d}{d \ln \mu} \left(\frac{m}{\alpha} \right) = -\frac{2m\beta(\alpha)}{\alpha^2} + \frac{m}{\alpha} \frac{d\beta(\alpha)}{d\alpha} = m\alpha \frac{d}{d\alpha} \left(\frac{\beta(\alpha)}{\alpha^2} \right). \quad (3)$$

Substituting the exact NSVZ β -function (2) into this equation we relate the expression in the left hand side to the anomalous dimension of the matter superfields:

$$\frac{d}{d \ln \mu} \left(\frac{m}{\alpha} \right) = -\frac{m\alpha N_f}{\pi} \cdot \frac{d\gamma(\alpha)}{d\alpha}. \quad (4)$$

The NSVZ relation was derived from general arguments (see, e.g., [10, 11, 12]) and, up to now, it is not completely clear in which subtraction scheme it is obtained. The subtraction scheme in which Eq. (1) is valid has not also been found. However, the NSVZ scheme has been constructed for (rigid) $\mathcal{N} = 1$ SQED with N_f flavors [13, 14, 15]. In order to specify this subtraction scheme one should regularize the theory by higher derivatives [16, 17]. (Unlike the dimensional reduction [18, 19], this method is mathematically consistent and can be formulated in a manifestly $\mathcal{N} = 1$ supersymmetric way [20, 21]. $\mathcal{N} = 2$ generalizations are also possible [22, 23, 24].) The main observation which allows constructing the NSVZ scheme is that, in the case of using the higher derivative regularization, the RG functions of $\mathcal{N} = 1$ SQED with N_f flavors defined in terms of the bare coupling constant (see Eq. (25) below) satisfy the NSVZ relation

$$\beta(\alpha_0) = \frac{\alpha_0^2 N_f}{\pi} \left(1 - \gamma(\alpha_0) \right) \quad (5)$$

in all orders independently of the subtraction scheme [25, 26]. The scheme independence follows from the fact [13] that these RG functions (defined in terms of the bare coupling constant) depend on a regularization, but do not depend on a subtraction scheme if a regularization is fixed. Eq. (5) follows from the factorization of integrals defining $\beta(\alpha_0)$ into integrals of (double) total derivatives [27, 28]. This feature of quantum corrections has been rigorously proved in all orders in [25, 26] and was confirmed by explicit three-loop calculations [29]. Factorization into double total derivatives was also demonstrated for various non-Abelian supersymmetric theories and for various versions of the higher derivative regularization [30, 31, 32, 33, 34, 35, 23, 24] in the lowest orders of the perturbation theory.

The scheme-dependence becomes essential for the RG functions defined (standardly) in terms of the renormalized coupling constant [36]. Starting from Eq. (5), it is possible to construct a simple prescription giving the subtraction scheme in which the RG functions defined in terms of the renormalized coupling constant satisfy the NSVZ relation in all orders [13, 14, 15], if the (Abelian) supersymmetric theory is regularized by higher derivatives.¹ Up to now, no analogs of this result have been found in the case of using the dimensional reduction, although structures similar to integrals of total derivatives were investigated [38]. At present, the only way to construct the NSVZ scheme with the dimensional reduction is making a specially tuned finite renormalization which relates it with the $\overline{\text{DR}}$ -scheme in every order of the perturbation theory [39, 40, 41, 42, 43]. The same situation takes place in theories with softly broken supersymmetry, because renormalization of the softly broken theories is related to renormalization of the rigid theories [44, 45, 46]. Thus, it is interesting to investigate what is the origin of Eq. (4), how it can be directly derived by summing Feynman diagrams, and in what subtraction scheme it is valid. In this paper we give the answers to the first two questions for softly broken $\mathcal{N} = 1$ SQED with N_f flavors, regularized by higher derivatives.

For this purpose we will use the method proposed in [25]. Subsequently, it was also applied for obtaining the exact expression for the Adler D -function [47] for $\mathcal{N} = 1$ SQCD in [48, 49].

¹A possible form of a similar prescription defining the NSVZ scheme for the non-Abelian supersymmetric theories regularized by higher covariant derivatives was discussed in [37].

In this paper we will demonstrate that this method can also be used in softly broken Abelian supersymmetric theories for deriving the exact expression for the photino mass anomalous dimension defined in terms of the bare coupling constant. In particular, we will prove that the renormalization of the photino mass is determined by integrals of double total derivatives in all orders.² Exactly as in the case of the rigid theory, such integrals do not vanish due to singularities of the integrands. By summing these singularities for softly broken $\mathcal{N} = 1$ SQED with N_f flavors regularized by higher derivatives it is possible to derive the exact equation

$$\frac{d}{d \ln \Lambda} \left(\frac{m_0}{\alpha_0} \right) = - \frac{m_0 \alpha_0 N_f}{\pi} \cdot \frac{d\gamma(\alpha_0)}{d\alpha_0}, \quad (6)$$

where m_0 is the bare photino mass and Λ denotes the dimensionful parameter of the regularized theory, which plays the role of the ultraviolet cut-off. (For the considered regularization it is valid in all orders independently of the subtraction scheme.) Differentiating the left hand side and using Eq. (5) we obtain the result for the anomalous dimension of the photino mass defined in terms of the bare coupling constant by the prescription

$$\gamma_m(\alpha_0) \equiv \frac{d \ln m_0}{d \ln \Lambda}, \quad (7)$$

which is written as

$$\gamma_m(\alpha_0) = \frac{\alpha_0 N_f}{\pi} \left[1 - \frac{d}{d\alpha_0} \left(\alpha_0 \gamma(\alpha_0) \right) \right]. \quad (8)$$

The paper is organized as follows: In Sect. 2 we introduce the higher derivative regularization for $\mathcal{N} = 1$ SQED with softly broken supersymmetry. In the next Sect. 3 we prove that integrals which determine the renormalization of the photino mass are integrals of double total derivatives by explicit summation of supergraphs in all orders. They are calculated in Sect. 4 by summing singularities of the integrands. The result gives Eq. (6) in all orders of the perturbation theory. This equation is verified by an explicit two-loop calculation in Sect. 5. In particular, we explicitly construct the integral of a double total derivative which defines the renormalization of the photino mass in the two-loop approximation. The results are briefly summarized in the Conclusion. Explicit expressions for various two-loop supergraphs are presented in Appendix.

2 Softly broken $\mathcal{N} = 1$ SQED and the higher derivative regularization

In this paper we consider softly broken $N = 1$ SQED with N_f flavours. It is described by the action

$$\begin{aligned} S = & \frac{1}{4e_0^2} \text{Re} \int d^4x d^2\theta (1 - 2m_0\theta^2) W^a W_a + \frac{1}{4} \sum_{f=1}^{N_f} \int d^4x d^4\theta (1 - \tilde{m}_{\phi 0}^2 \theta^4) \left(\phi_f^* e^{2V} \phi_f \right. \\ & \left. + \tilde{\phi}_f^* e^{-2V} \tilde{\phi}_f \right) + \sum_{f=1}^{N_f} \left(\frac{1}{2} \int d^4x d^2\theta m_{\phi 0} (1 - \theta^2 b_0) \phi_f \tilde{\phi}_f + \text{c.c.} \right), \end{aligned} \quad (9)$$

where V denotes the supersymmetric gauge superfield with the strength $W_a = \bar{D}^2 D_a V / 4$; ϕ_f and $\tilde{\phi}_f$ are chiral matter superfields. The bare coupling constant is denoted by e_0 , and $m_{\phi 0}$ is

²Although the technique proposed in [7] allows to relate the results of calculations in a softly broken theory to the ones in the rigid theory, it would be interesting to derive the exact result by a straightforward calculation.

the bare mass of the matter superfields in the rigid theory. The soft breaking parameter m_0 is the bare photino mass; $\tilde{m}_{\phi 0}$ and b_0 are also the bare soft breaking parameters with the dimension of mass. The soft breaking terms contain the spurion $\eta \equiv \theta^2$.³ In our notation,

$$\theta^2 \equiv \theta^a \theta_a = \eta; \quad \bar{\theta}^2 \equiv \bar{\theta}^{\dot{a}} \bar{\theta}_{\dot{a}} = \bar{\eta}; \quad \theta^4 \equiv \theta^2 \bar{\theta}^2 = \eta \bar{\eta}. \quad (10)$$

The considered theory is invariant under $U(1)$ gauge transformations. Due to this symmetry, terms linear and cubic in the matter superfields are forbidden.

In this paper we are interested in the renormalization of the photino mass m in the limit when the parameters m_ϕ , \tilde{m}_ϕ , and b vanish. This implies that only the following terms in the action will be essential below:

$$S \rightarrow \frac{1}{4e_0^2} \text{Re} \int d^4x d^2\theta (1 - 2m_0\theta^2) W^a W_a + \frac{1}{4} \sum_{f=1}^{N_f} \int d^4x d^4\theta \left(\phi_f^* e^{2V} \phi_f + \tilde{\phi}_f^* e^{-2V} \tilde{\phi}_f \right). \quad (11)$$

As for m , we will consider only terms linear in m and neglect m -dependence of the Green functions. This means that investigating the renormalization of various Green functions we consider the limit in which m is much smaller than the external momenta.

For calculating quantum corrections in the considered theory we will use the higher derivative regularization. It can be introduced by adding the higher derivative term

$$S_\Lambda = \frac{1}{4e_0^2} \text{Re} \int d^4x d^2\theta (1 - 2m_0\theta^2) W^a \left(R(\partial^2/\Lambda^2) - 1 \right) W_a \quad (12)$$

to the action, where Λ is a parameter with the dimension of mass, and $R(x)$ is a regulator rapidly growing at infinity and satisfying the condition $R(0) = 1$. For example, one can choose $R(x) = 1 + x^n$, where n is an integer. It is convenient to choose the gauge fixing term

$$S_{\text{gf}} = -\frac{1}{32\xi_0 e_0^2} \int d^4x d^4\theta (1 - m_0\theta^2 - m_0\bar{\theta}^2) D^2 V K(\partial^2/\Lambda^2) \bar{D}^2 V, \quad (13)$$

where K is a function which rapidly grows at infinity and, by definition, $K(0) = 1$. Then the terms quadratic in the gauge superfield can be written as

$$\begin{aligned} & -\frac{1}{4e_0^2} \int d^4x d^4\theta \left\{ (1 - m_0\theta^2 - m_0\bar{\theta}^2) V \left[R\partial^2 \Pi_{1/2} + K \frac{1}{16\xi_0} (\bar{D}^2 D^2 + D^2 \bar{D}^2) \right] V \right. \\ & \left. + \frac{m_0}{4} V R \cdot \left[\theta^a \bar{D}^2 D_a V + \bar{\theta}^{\dot{a}} D^2 \bar{D}_{\dot{a}} \right] V - \frac{m_0}{4\xi_0} V K \cdot \left[\theta^a D_a \bar{D}^2 + \bar{\theta}^{\dot{a}} \bar{D}_{\dot{a}} D^2 - \bar{D}^2 - D^2 \right] V \right\}. \quad (14) \end{aligned}$$

Note that the terms in the second string do not contain the second degree of θ -s and come from the derivatives of the spurion. We will calculate quantum corrections by using the supergraph method [50], which can be also used for theories with softly broken supersymmetry [51]. We will consider the terms in the first string as the free action. The terms in the second string will be treated as the interaction. They give vertices in which external lines correspond to the first degree θ polynomials proportional to m_0 . It is well known that these vertices do not contribute to the considered RG function [51]. This can also be seen from the calculations made in this paper. According to [52, 53, 54] the propagator of the gauge superfield obtained from Eq. (14) is proportional to⁴

³Below in this paper we prefer to write θ -s instead of η and $\bar{\eta}$.

⁴Constructing this expression one does not take into account the terms in the second string of Eq. (14), which are considered as interaction.

$$e_0^2(1+m_0\theta^2+m_0\bar{\theta}^2)\left[-\frac{1}{R\partial^2}+\frac{1}{16\partial^4}\left(\frac{\xi_0}{K}-\frac{1}{R}\right)\left(\bar{D}^2D^2+D^2\bar{D}^2\right)\right]\delta_{xy}^8+O(m_0\theta,m_0\bar{\theta})+O(m_0^2). \quad (15)$$

In this equation we omit the terms proportional to $m_0\theta$, $m_0\bar{\theta}$, and m_0 without θ -s (denoted by $O(m_0\theta,m_0\bar{\theta})$) and all terms proportional to $(m_0)^n$ with $n \geq 2$ (denoted by $O(m_0^2)$). Below we will see that they do not affect the considered RG functions. The propagator (15) contains large degrees of the momentum in the denominator inside the functions R and K . Consequently, all diagrams beyond one-loop approximation become finite (at finite values of Λ). However, divergences can be present in the one-loop graphs [55]. According to [56], these remaining one-loop divergences are regularized by inserting the Pauli–Villars determinants

$$\begin{aligned} \text{Det}(PV, V, M_I)^{-1} &\equiv \int D\Phi_I D\tilde{\Phi}_I \exp\left(\frac{i}{4} \int d^4x d^4\theta \left(\Phi_I^* e^{2V} \Phi_I + \tilde{\Phi}_I^* e^{-2V} \tilde{\Phi}_I\right)\right) \\ &+ \frac{i}{2} \int d^4x d^2\theta M_I \Phi_I \tilde{\Phi}_I + \frac{i}{2} \int d^4x d^2\bar{\theta} M_I \Phi_I^* \tilde{\Phi}_I^* \end{aligned} \quad (16)$$

into the generating functional. (The superfields Φ_I and $\tilde{\Phi}_I$ are commuting.) It is important that the Pauli–Villars masses M_I should be proportional to the parameter Λ in the higher derivative term S_Λ , the coefficients of the proportionality being independent of the coupling constant. Taking into account that in the Abelian case it is not necessary to introduce the Faddeev–Popov ghosts, the generating functional for the connected Green functions can be presented in the form

$$W = -i \ln \int DV D\phi D\tilde{\phi} \prod_{I=1}^m \text{Det}(PV, V, M_I)^{N_I c_I} \exp\left(iS + iS_\Lambda + iS_{\text{gf}} + iS_{\text{Sources}}\right). \quad (17)$$

The coefficients c_I are introduced in order to cancel the remaining one-loop divergencies. They satisfy the equation $\sum_{I=0}^m c_I = 0$, where, by definition, $c_0 \equiv -1$. The action for sources is written as

$$S_{\text{Sources}} = \int d^4x d^4\theta VJ + \left(\int d^4x d^2\theta (\phi j + \tilde{\phi} \tilde{j}) + \text{c.c.}\right). \quad (18)$$

The effective action $\Gamma[\mathbf{V}, \phi, \tilde{\phi}]$ is obtained from the functional W by making the Legendre transformation. Note that, for later convenience, we denote the argument of the effective action corresponding to the gauge superfield by the bold letter \mathbf{V} . This is done in order to distinguish it from the integration variable V in the generating functional.

The NSVZ-like equation (6) relates divergencies in the two-point Green function of the gauge superfield and of the matter superfields. Taking into account the gauge invariance of the action and the gauge invariance of the regularization one can verify that the quantum corrections to the former Green function should be transversal due to the Ward identity. Consequently, the part of the effective action corresponding to this two-point Green function can be presented in the form [51]

$$\begin{aligned} \Gamma_{\mathbf{V}}^{(2)} - S_{\text{gf}} &= -\frac{1}{16\pi} \int \frac{d^4p}{(2\pi)^4} d^4\theta \left(\mathbf{V}(-p, \theta) \partial^2 \Pi_{1/2} \mathbf{V}(p, \theta) d^{-1}(\alpha_0, \Lambda/p, \dots) \right. \\ &\left. - \frac{m_0}{8} \left(\theta^2 D^a \mathbf{V}(-p, \theta) \bar{D}^2 D_a \mathbf{V}(p, \theta) + \bar{\theta}^2 \bar{D}^{\dot{a}} \mathbf{V}(-p, \theta) D^2 \bar{D}_{\dot{a}} \mathbf{V}(p, \theta) \right) d_m^{-1}(\alpha_0, \Lambda/p, \dots) \right), \end{aligned} \quad (19)$$

where dots denote the other arguments of the (dimensionless) functions d and d_m , which include various mass parameters. Note that we do not include in this expression a term proportional to $\int d^4\theta \mathbf{V}$, because in the considered theory it is forbidden by the Z_2 -symmetry $V \rightarrow -V$; $\phi \leftrightarrow \tilde{\phi}$. The normalization constants in Eq. (19) are chosen so that in the tree approximation $d^{-1} = \alpha_0^{-1} + O(1)$ and $d_m^{-1} = \alpha_0^{-1} + O(1)$.

The two-point Green function of the chiral matter superfields for theories with softly broken supersymmetry is θ -dependent and can be written as

$$\begin{aligned} & \frac{1}{4} \sum_{f=1}^{N_f} \int \frac{d^4 p}{(2\pi)^4} d^4\theta \left(\phi_f^*(-p, \theta) \phi_f(p, \theta) + \tilde{\phi}_f^*(-p, \theta) \tilde{\phi}_f(p, \theta) \right) \left(G(\alpha_0, \Lambda/p, \dots) \right. \\ & \left. + m_0 \theta^2 g(\alpha_0, \Lambda/p, \dots) + m_0 \bar{\theta}^2 g^*(\alpha_0, \Lambda/p, \dots) + m_0^2 \theta^4 \tilde{g}(\alpha_0, \Lambda/p, \dots) \right). \end{aligned} \quad (20)$$

The terms linear in the chiral matter superfields are evidently forbidden in the considered theory.

We will be interested in the behaviour of the function d_m^{-1} in the limit when all massive parameters (except for Λ and, therefore, M_I) tend to 0. In this limit

$$d^{-1}(\alpha_0, \Lambda/p, \dots) \rightarrow d^{-1}(\alpha_0, \Lambda/p); \quad d_m^{-1}(\alpha_0, \Lambda/p, \dots) \rightarrow d_m^{-1}(\alpha_0, \Lambda/p). \quad (21)$$

Similarly, the two-point function of the chiral matter superfields for the massless rigid theory in this limit can be presented in the form

$$G(\alpha_0, \Lambda/p, \dots) \rightarrow G(\alpha_0, \Lambda/p). \quad (22)$$

The renormalized coupling constant $\alpha(\alpha_0, \Lambda/\mu)$ and the renormalization constant for the photino mass $Z_m(\alpha, \Lambda/\mu)$, where μ is a normalization point, are defined by requiring finiteness of the functions

$$d^{-1}(\alpha_0(\alpha, \Lambda/\mu), \Lambda/p) \quad \text{and} \quad Z_m(\alpha, \Lambda/\mu) d_m(\alpha_0(\alpha, \Lambda/\mu), \Lambda/p), \quad (23)$$

respectively. (The second equation implies that the renormalized photino mass is related to the bare one as $m = Z_m m_0$.) The renormalization constant for the chiral matter superfields becomes θ -dependent:

$$\phi = \mathcal{Z}(\alpha, \Lambda/\mu, \theta^2) \phi_R \equiv \sqrt{Z(\alpha, \Lambda/\mu)} \left(1 + m_0 \theta^2 z(\alpha, \Lambda/\mu) \right) \phi_R. \quad (24)$$

The renormalization constant Z is defined in such a way that the expression ZG considered as a function of the renormalized coupling constant is finite. Renormalization of the other parameters is not considered in this paper.

In this paper we investigate the RG functions defined in terms of the bare coupling constant as

$$\beta(\alpha_0) = \left. \frac{d\alpha_0}{d \ln \Lambda} \right|_{\alpha=\text{const}}; \quad \gamma(\alpha_0) = - \left. \frac{d \ln Z}{d \ln \Lambda} \right|_{\alpha=\text{const}}; \quad \gamma_m(\alpha_0) = - \left. \frac{d \ln Z_m}{d \ln \Lambda} \right|_{\alpha=\text{const}}, \quad (25)$$

where the differentiation with respect to $\ln \Lambda$ should be made at a fixed value of the renormalized coupling constant. (The last equation is evidently equivalent to Eq. (7).) As we have already mentioned above, they depend only on the regularization, but are not changed under finite renormalizations (for a fixed regularization).

3 Integrals of double total derivatives which determine renormalization of the photino mass

3.1 Two-point Green function of the gauge superfield

To obtain the NSVZ-like result for the renormalization of the photino mass in Abelian supersymmetric theories by summing supergraphs, it is possible to use the idea proposed in [25]. It is based on the observation that in this case the action is quadratic in the chiral matter superfields. This implies that the functional integrals over these matter superfields can formally be calculated exactly in all orders. Following Ref. [25], the result can be presented in the form

$$\int D\phi D\tilde{\phi} \exp \left\{ i \left[\frac{1}{4} \int d^4x d^4\theta \left(\phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) + \left(\int d^4x d^2\theta \left(\frac{1}{2} M \tilde{\phi} \phi + j\phi + \tilde{j}\tilde{\phi} \right) + \text{c.c.} \right) \right] \right\} = \det(\star)^{1/2} \cdot \exp \left(\frac{i}{2} \int d^4x d^4\theta (Aj)^T P \star Aj \right). \quad (26)$$

In this equation the currents are present in the combination

$$Aj \equiv \frac{1}{4\partial^2} \begin{pmatrix} D^2 j \\ \bar{D}^2 j^* \\ D^2 \tilde{j} \\ \bar{D}^2 \tilde{j}^* \end{pmatrix}, \quad (27)$$

corresponding to the sequence $(\phi, \phi^*, \tilde{\phi}, \tilde{\phi}^*)$ of the matter superfields. The operator

$$\star \equiv (1 - I_0)^{-1}, \quad \text{with} \quad I_0 \equiv BP, \quad (28)$$

encodes chains of the vertices B and the propagators P of arbitrary lengths. The vertices are included into the matrix

$$B \equiv \begin{pmatrix} 0 & e^{2V} - 1 & 0 & 0 \\ e^{2V} - 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-2V} - 1 \\ 0 & 0 & e^{-2V} - 1 & 0 \end{pmatrix}, \quad (29)$$

while the propagator matrix has the form

$$P = \begin{pmatrix} 0 & \frac{\bar{D}^2 D^2}{16(\partial^2 + M^2)} & \frac{M \bar{D}^2}{4(\partial^2 + M^2)} & 0 \\ \frac{D^2 \bar{D}^2}{16(\partial^2 + M^2)} & 0 & 0 & \frac{M D^2}{4(\partial^2 + M^2)} \\ \frac{M \bar{D}^2}{4(\partial^2 + M^2)} & 0 & 0 & \frac{\bar{D}^2 D^2}{16(\partial^2 + M^2)} \\ 0 & \frac{M D^2}{4(\partial^2 + M^2)} & \frac{D^2 \bar{D}^2}{16(\partial^2 + M^2)} & 0 \end{pmatrix}. \quad (30)$$

Using Eq. (26) the generating functional for the connected Green functions can be presented in the form

$$W = -i \ln \int DV \prod_{I=0}^m \text{Det}(PV, V, M_I)^{N_{fcI}} \exp \left(i(S_{\text{gauge}} + S_{\Lambda} + S_{\text{gf}}) + i \int d^8x JV \right) \times \exp \left(\frac{i}{2} \int d^8x (Aj)^T P \star Aj \right), \quad (31)$$

where

$$S_{\text{gauge}} = \frac{1}{4e_0^2} \text{Re} \int d^4x d^2\theta (1 - 2m_0\theta^2) W^a W_a, \quad (32)$$

and $I = 0$ stands for the original superfields ϕ and $\tilde{\phi}$, so that $M_0 = 0$ and $c_0 = -1$.

Starting from the expression (31) and repeating the calculations made in [25] we obtain the following expression for the part of the effective action corresponding to the two-point Green function of the gauge superfield:

$$\begin{aligned} \Delta\Gamma_{\mathbf{V}}^{(2)} \equiv \Gamma_{\mathbf{V}}^{(2)} - S_{\mathbf{V}}^{(2)} - S_{\text{gf}} = & -\frac{i}{2} N_f^2 \left\langle \left(\sum_{I=0}^m c_I \text{Tr}(\mathbf{V} Q J_0 \star) \right)_I^2 \right\rangle_{\text{1PI}} \\ & + i N_f \sum_{I=0}^m c_I \text{Tr} \left\langle \mathbf{V} Q J_0 \star \mathbf{V} Q J_0 \star + \mathbf{V}^2 J_0 \star \right\rangle_{I, \text{1PI}}, \end{aligned} \quad (33)$$

where the argument of the effective action is denoted by the bold letter \mathbf{V} in order to distinguish it from the integration variable V . The angular brackets are defined by the prescription

$$\langle A(V) \rangle \equiv \frac{1}{Z} \int DV A(V) \prod_{I=0}^m \text{Det}(PV, V, M_I)^{N_f c_I} \exp \left(i(S_{\text{gauge}} + S_{\Lambda} + S_{\text{gf}}) + i \int d^8x J V \right), \quad (34)$$

where $Z = \exp(iW)$, and the source J should be expressed in terms of \mathbf{V} in the standard way by making the Legendre transformation. The subscripts 1PI point that it is necessary to take into consideration only one-particle irreducible graphs. Also we use the notation

$$Q \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}; \quad J_0 = \begin{pmatrix} 0 & e^{2V} & 0 & 0 \\ e^{2V} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-2V} \\ 0 & 0 & e^{-2V} & 0 \end{pmatrix} P. \quad (35)$$

The expression (33) has a simple graphical interpretation. The first term corresponds to diagrams in which external lines are attached to different loops of the matter superfields. Usually, it is called the singlet contribution. Below we prove that for the considered theory this contribution vanishes. The second term can be interpreted as a sum of diagrams in which the external \mathbf{V} -lines are attached to different points of a single matter loop. The last term encodes the sum of diagrams in which external lines are attached to a single point on a matter line.

Although Eq. (33) looks exactly as a similar expression presented in [25] for the rigid theory, there is an essential difference. Namely, the angular brackets are now defined in a different way, because the action $S_{\text{gauge}} + S_{\Lambda}$ contains the soft photino mass term. In particular, this implies that the angular brackets introduce explicit dependence on θ . Therefore, the effective supergraphs which are obtained from Eq. (31) should be calculated using somewhat different rules in comparison with the rigid theory.

It is convenient to rewrite the expression (33) in a different form. The corresponding calculation has been done in [49]. Here we only present the result and its graphical interpretation.

The first term in Eq. (33) contains the expression $\text{Tr}(\star \mathbf{V} Q J_0)$, which can be equivalently presented in the form

$$\text{Tr}(\star \mathbf{V} Q J_0) = \text{Tr} \left\{ \star \left(B P(\mathbf{V} Q) B_0 P + B(\mathbf{V} Q)(\Pi_+ P) + B(P \Pi_-)(\mathbf{V} Q) \right) \right\}, \quad (36)$$

where the chiral projection operators have the form

$$\Pi_+ \equiv -\frac{\bar{D}^2 D^2}{16\partial^2}; \quad \Pi_- \equiv -\frac{D^2 \bar{D}^2}{16\partial^2}, \quad (37)$$

and

$$B_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad (38)$$

encodes the vertices with a single external \mathbf{V} -line attached to the matter line (and no internal V -lines). The sum in the round brackets can be interpreted as a sum of the subdiagrams presented in Fig. 1, which were first considered in [25]. The operator \star gives a sequence of vertices and propagators of an arbitrary length, and Tr converts this chain to a closed loop. The angular brackets in Eq. (33) make internal gauge lines from all V -s in the product of two expressions (36), giving the singlet contribution to the considered two-point function.

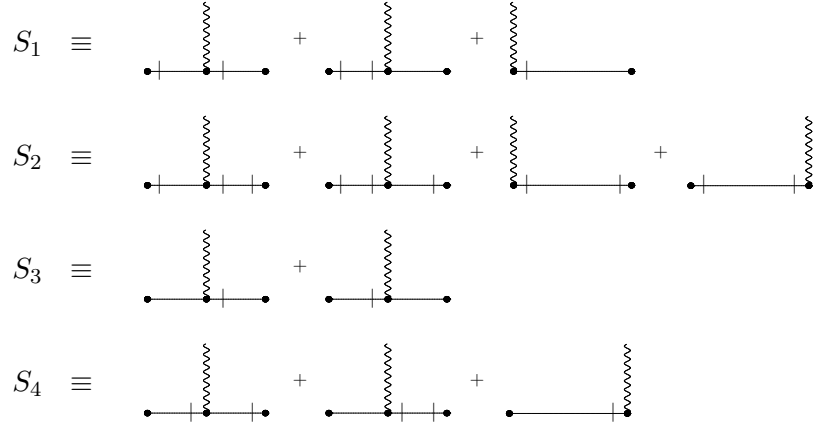


Figure 1: Elements of the matrix $BP(\mathbf{V}Q)B_0P + B(\mathbf{V}Q)(\Pi_+P) + B(P\Pi_-)(\mathbf{V}Q)$ correspond to the sum of subdiagrams presented in this figure.

According to [49] the second term in Eq. (33) can be presented as a sum of three terms,

$$\text{Tr}\langle \star \mathbf{V}QJ_0 \star \mathbf{V}QJ_0 \rangle = A_0 + A_1 + A_2 \quad (39)$$



Figure 2: Graphical interpretation of various terms in Eq. (39).

containing different numbers of the operator \star . They can be graphically interpreted as the effective diagrams presented in Fig. 2. Namely,

$$A_0 \equiv \text{Tr}\left((\mathbf{V}QB_0)P(\mathbf{V}QB_0)P\right) \quad (40)$$

does not contain the operator \star (and, consequently, the angular brackets). It gives the one-loop contribution to the considered two-point function. Therefore, the one-loop approximation in this approach should be considered separately.

The expression

$$A_1 = 2 \cdot \text{Tr} \left\langle \star \left(BP(\mathbf{V}QB_0)P(\mathbf{V}QB_0)P + (\mathbf{V}BQ)(\Pi_+P)(\mathbf{V}QB_0)P \right. \right. \\ \left. \left. + BP(\mathbf{V}QB_0)(P\Pi_-)(\mathbf{V}Q) + (B\mathbf{V}Q)(\Pi_+P\Pi_-)(\mathbf{V}Q) \right) \right\rangle \quad (41)$$

is constructed from the terms containing a single operator \star . Graphically, it can be presented as the second effective diagram in Fig. 2. The effective vertex in this diagram consists of a large number of subdiagrams with two external \mathbf{V} -lines. They are presented in Fig. 3.

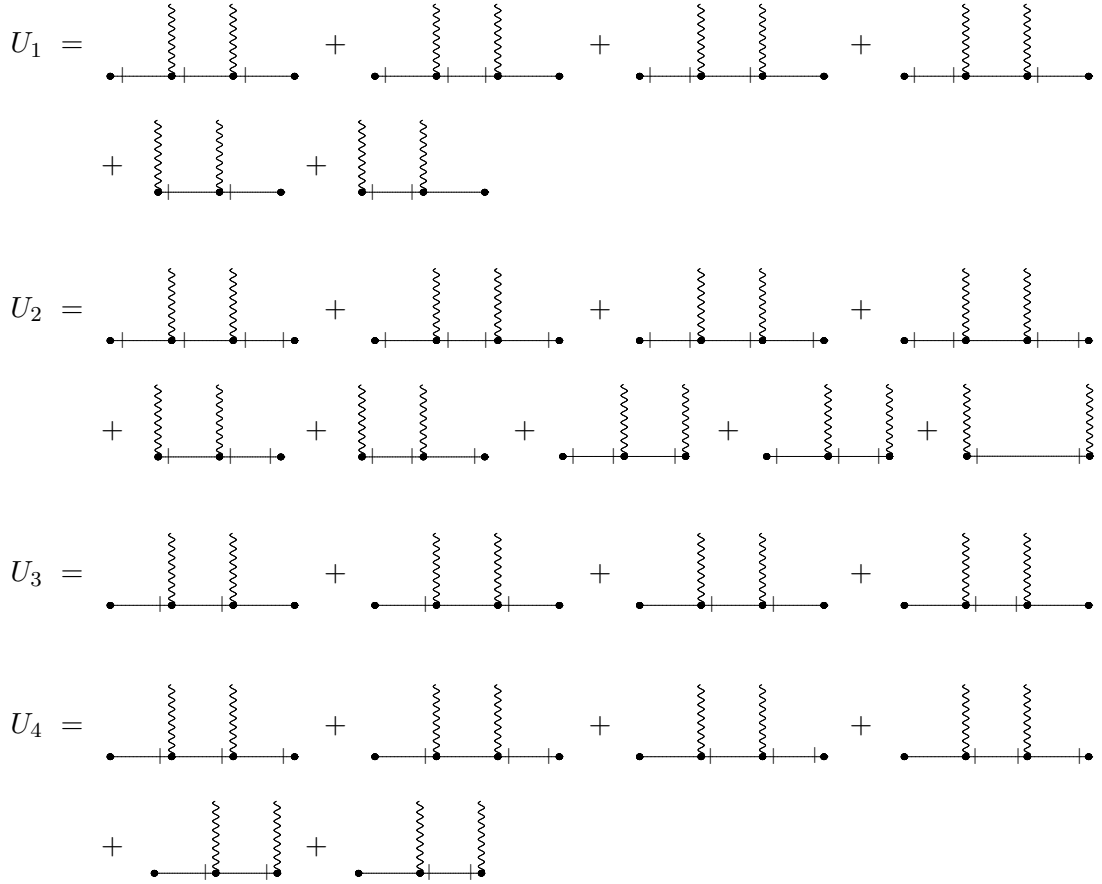


Figure 3: Subdiagrams corresponding to the effective vertex in the effective diagram A_1 presented in Fig. 2. Note that various options of the end chiralities lead to four different sums of subdiagrams.

The last term in Eq. (39) is a sum of contributions containing two operators \star . It is explicitly written as

$$A_2 = \text{Tr} \left\langle \star \left(BP(\mathbf{V}Q)B_0P + B(\mathbf{V}Q)(\Pi_+P) + B(P\Pi_-)(\mathbf{V}Q) \right) \right. \\ \left. \star \left(BP(\mathbf{V}Q)B_0P + B(\mathbf{V}Q)(\Pi_+P) + B(P\Pi_-)(\mathbf{V}Q) \right) \right\rangle \quad (42)$$

and can be graphically presented as the third diagram in Fig. 2. Each of two effective vertices in this diagram is given by the sum of subdiagrams presented in Fig. 1. As usual, the operators \star and Tr make the closed loop, and the angular brackets transform V into gauge propagators.

Below we will see that the last term in Eq. (33) (which contains V^2) is not essential in calculating the considered RG function, because it gives the vanishing contribution. That is why we do not discuss in details its structure.

To calculate the left hand side of Eq. (6), it is convenient to consider the expression

$$\frac{d}{d \ln \Lambda} \left(\frac{m_0}{d_m(\alpha_0, \Lambda/p)} - \frac{m_0}{\alpha_0} \right) \Big|_{p=0} = - \frac{d}{d \ln \Lambda} \left(\frac{m_0}{\alpha_0} \right), \quad (43)$$

where the differentiation is made at fixed values of the renormalized coupling constant and of the renormalized photino mass. The equality follows from the finiteness of the expression $m_0 d_m^{-1} = m(Z_m d_m)^{-1}(\alpha, \mu/p, \Lambda/p)$. The limit $p \rightarrow 0$ should be taken in order to get rid of the various terms proportional to $(p/\Lambda)^k$.

The left hand side of Eq. (43) can be obtained from the part of $\Delta\Gamma = \Gamma - S - S_{\text{gf}}$ corresponding to the two-point Green function of the gauge superfield. We denote it by $\Delta\Gamma_{\mathbf{V}}^{(2)}$. By other words, $\Delta\Gamma_{\mathbf{V}}^{(2)}$ is a part of $\Delta\Gamma$ which contains only terms quadratic in the gauge superfield \mathbf{V} . To extract the expression (43), it is possible to make the formal substitution

$$\mathbf{V} \rightarrow \bar{\theta}^{\dot{a}} \bar{\theta}_{\dot{a}} \theta^b \psi_b \equiv \bar{\theta}^2 \theta^b \psi_b, \quad (44)$$

where ψ_b is a slowly changing spinor which is approximately constant at finite x^μ and tends to 0 at some large scale $R \rightarrow \infty$. Really, after this substitution the part of the gauge superfield two-point function corresponding to the rigid theory (which contains the function d^{-1}) vanishes, because the supersymmetric transversal projection operator $\partial^2 \Pi_{1/2}$ contains four spinor derivatives. Note that the derivatives ∂_μ acting on ψ give terms suppressed as $1/R\Lambda$, because ψ is almost constant. These terms are negligible in the limit $R \rightarrow \infty$ and should be omitted. However, the part of the gauge superfield two-point function containing the function d_m^{-1} survives after the substitution (44) and, after some transformations, gives

$$\frac{d\Delta\Gamma_{\mathbf{V}}^{(2)}}{d \ln \Lambda} \Big|_{\mathbf{V}=\bar{\theta}^2 \theta^b \psi_b} = - \frac{1}{8\pi} \mathcal{V}_\psi \cdot \frac{d}{d \ln \Lambda} \left(\frac{m_0}{d_m(\alpha_0, \Lambda/p)} - \frac{m_0}{\alpha_0} \right) \Big|_{p=0}, \quad (45)$$

where we introduced the notation

$$\mathcal{V}_\psi \equiv \int d^4 x \psi^a \psi_a \sim R^4 \rightarrow \infty. \quad (46)$$

(Note that the condition $p \rightarrow 0$ appears, because the spinor ψ_b is almost constant. It follows from the fact that the external momentum p has the order $1/R$ according to the definition of the spinor ψ .) It is important that the only non-vanishing (after the substitution (44)) term in Eq. (19) is the one containing $m_0 \theta^2$, while the term containing $m_0 \bar{\theta}^2$ vanishes. That is why it is also possible to set formally

$$m_0 \bar{\theta}^2 \rightarrow 0 \quad (47)$$

in S_{gauge} and S_Λ before the substitution (44), as we will always do below. (However, $m_0 \theta^2 \neq 0$.)

3.2 Factorization into double total derivatives

Let us calculate the expression (45) by using Eq. (33). As we have demonstrated in the previous section, the result can be obtained by calculating some effective diagrams which contain

certain subdiagrams. That is why we start with the calculation of the subdiagrams presented in Fig. 1, in which we make the substitution (44). The analytical expressions for them are encoded in the expression

$$BP(\mathbf{V}Q)B_0P + B(\mathbf{V}Q)(\Pi_+P) + B(P\Pi_-)(\mathbf{V}Q) = \begin{pmatrix} (e^{2V}-1)S_4 & 0 & 0 & (e^{2V}-1)S_3 \\ 0 & (e^{2V}-1)S_1 & (e^{2V}-1)S_2 & 0 \\ 0 & -(e^{-2V}-1)S_3 & -(e^{-2V}-1)S_4 & 0 \\ -(e^{-2V}-1)S_2 & 0 & 0 & -(e^{-2V}-1)S_1 \end{pmatrix}, \quad (48)$$

where S_1 , S_2 , S_3 , and S_4 denote expressions for the subdiagrams presented in Fig. 1. Omitting the derivatives of the spinor ψ , which vanish in the limit $p \rightarrow 0$, after some transformations the result can be written as

$$\begin{aligned} S_1 &= -\bar{\theta}^{\dot{a}}\theta^b\psi_b \frac{\bar{D}_{\dot{a}}D^2}{4(\partial^2+M^2)} + i\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\psi_b \frac{\bar{D}^2D^2\partial_\mu}{16(\partial^2+M^2)^2} + \text{terms without } \bar{\theta} \\ &= \frac{i}{2}\bar{\theta}\gamma^\mu\psi\left[y_\mu^*, \frac{\bar{D}^2D^2}{16(\partial^2+M^2)}\right] + \text{terms without } \bar{\theta}; \\ S_2 &= -\bar{\theta}^{\dot{a}}\theta^b\psi_b \frac{M\bar{D}_{\dot{a}}}{\partial^2+M^2} + i\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\psi_b \frac{M\bar{D}^2\partial^\mu}{4(\partial^2+M^2)^2} + \text{terms without } \bar{\theta} \\ &= \frac{i}{2}\bar{\theta}\gamma^\mu\psi\left[y_\mu^*, \frac{M\bar{D}^2}{4(\partial^2+M^2)}\right] + \text{terms without } \bar{\theta}; \\ S_3 &= -i\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\psi_b \frac{MD^2\partial_\mu}{4(\partial^2+M^2)^2} + \text{terms without } \bar{\theta} \\ &= -\frac{i}{2}\bar{\theta}\gamma^\mu\psi\left[y_\mu^*, \frac{MD^2}{4(\partial^2+M^2)}\right] + \text{terms without } \bar{\theta}; \\ S_4 &= -i\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\psi_b \frac{D^2\bar{D}^2\partial_\mu}{16(\partial^2+M^2)^2} + \bar{\theta}^{\dot{a}}\theta^b\psi_b \frac{D^2\bar{D}_{\dot{a}}}{4(\partial^2+M^2)} + \bar{\theta}^{\dot{a}}\psi^b \frac{D_b\bar{D}_{\dot{a}}}{2(\partial^2+M^2)} \\ &\quad + \text{terms without } \bar{\theta} = -\frac{i}{2}\bar{\theta}\gamma^\mu\psi\left[y_\mu^*, \frac{D^2\bar{D}^2}{16(\partial^2+M^2)}\right] + \text{terms without } \bar{\theta}, \end{aligned} \quad (49)$$

where $\bar{\theta}\gamma^\mu\psi \equiv \bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\psi_b$. M is equal either to 0 for the usual matter superfields, or to M_I for the Pauli–Villars superfields. The (anti)chiral coordinates are defined by

$$y^\mu = x^\mu + i\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\theta_b; \quad (y^\mu)^* = x^\mu - i\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\theta_b. \quad (50)$$

Below we will see that the terms without $\bar{\theta}$ do not contribute to the considered RG function (which determines the running photino mass). Looking at the expressions (49) we see that all terms in the right hand side contain commutators of y_μ^* with various components of the matrix P . Therefore, from Eqs. (48) and (49) we obtain

$$BP(\mathbf{V}Q)B_0P + B(\mathbf{V}Q)(\Pi_+P) + B(P\Pi_-)(\mathbf{V}Q) = \frac{i}{2}\bar{\theta}\gamma^\mu\psi[y_\mu^*, \tilde{Q}I_0] + \text{terms without } \bar{\theta}, \quad (51)$$

where

$$\tilde{Q} \equiv \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (52)$$

(It is easy to see that $[\tilde{Q}, I_0] = 0$ and $[\tilde{Q}, \star] = 0$.) Then we make the transformations similar to the ones in [25, 49], taking into account that trace of the commutator with $-i\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\theta_b$ vanishes, so that y_μ^* can be replaced by x_μ . As a result, we obtain

$$\text{Tr}(\mathbf{V}QJ_0\star)\Big|_{\mathbf{V}=\bar{\theta}^2\theta^a\psi_a} = \frac{i}{2}\text{Tr}\left(\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\psi_b\tilde{Q}[x_\mu, \ln\star]\right) + \dots, \quad (53)$$

where dots denote terms which do not contain $\bar{\theta}$. The commutator with x^μ in the momentum representation gives the derivative with respect to the loop momentum, and Tr gives the integration over this loop momentum. Therefore, we obtain that loop integrals encoded in Eq. (53) are integrals of total derivatives.

According to Eq. (33), the sum of diagrams in which the external lines are attached to different matter loops contains two traces (53). The terms without $\bar{\theta}$ do not contribute to such diagrams, because the nontrivial result can be obtained only if θ^4 is present in a supergraph. (Let us remind that we have set $\bar{\theta}^2 = 0$ in S_{gauge} and S_Λ .) Thus, the considered part of the effective action proportional to N_f^2 can be presented in the form of an integral of a double total derivative

$$\frac{i}{8}N_f^2\left\langle\left(\text{Tr}\sum_{I=0}^m c_I\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\psi_b\tilde{Q}[x_\mu, \ln(\star_I)]\right)^2\right\rangle. \quad (54)$$

More exactly, each matter loop to which an external line is attached gives an integral over a total derivative. Evidently, all diagrams in the considered contribution contain two such loops. It is important that there are no singularities in Eq. (54), which can produce δ -functions. (We will discuss such singularities in detail below.)

The main difference of Eq. (54) from the corresponding expressions in Refs. [25, 49] is that it contains only $\bar{\theta}^2$ instead of θ^4 in the rigid theory. In the rigid theory all contribution which contain $\bar{\theta}^2$ and do not contain θ^2 vanish. Really, explicit θ -dependence cannot appear in calculating supergraphs by using the algebra of supersymmetric covariant derivatives. Therefore, any non-vanishing superdiagram should contain θ^4 due to the integration over $d^4\theta$.

However, the situation is different in the softly broken theory, because the gauge propagators (15) and vertices coming from the second string of Eq. (14) explicitly depend on θ . Therefore, θ^2 is introduced by the functional integration over the gauge superfield which is denoted by the angular brackets. Evidently, all terms containing θ^2 are proportional at least to the first degree of m_0 . This implies that the expression (54) does not vanish due to the integration over $d^4\theta$. However, it certainly vanishes as a trace of a commutator.

Now, let us proceed to calculating the terms proportional to N_f in Eq. (33). First, we note that the last term proportional to \mathbf{V}^2 vanishes after the substitution (44) due to anticommutation of $\bar{\theta}$ -components ($\bar{\theta}^2 \cdot \bar{\theta}^2 = 0$). Certainly, this result is quite reasonable, because this term is not transversal. Such terms cancel each other due to the Ward identity and do not affect the RG functions considered in this paper.

Thus, there is the only non-vanishing term in Eq. (33) which is proportional to the expression (39). As we saw above, it can be written as a sum of three contributions A_0 , A_1 , and A_2 , which are graphically presented in Fig. 2. It is easy to see that the one-loop contribution A_0 vanishes after the substitution (44),

$$A_0\Big|_{\mathbf{V}=\bar{\theta}^2\theta^a\psi_a} = \text{One-loop} = 0. \quad (55)$$

Really, the expression A_0 is determined by the diagram without internal lines of the gauge superfield. Therefore, m_0 does not enter in this supergraph. All other massive parameters (except for the regularization parameter Λ) are set to 0. Therefore, this diagram is exactly the

same as in the rigid theory and contains contributions proportional to $\int d^4\theta \mathbf{V} \partial \Pi_{1/2} \mathbf{V}$ and to $\int d^4\theta \mathbf{V}^2$. It is easy to see that both these expressions vanish after the substitution (44).

The expression A_1 is graphically presented in Fig. 2, where it corresponds to the second effective diagram. The effective vertex in this diagram is given by the expression in the round brackets in Eq. (41). This expression encodes the sums of vertices presented in Fig. 3:

$$BP(\mathbf{V}QB_0)P(\mathbf{V}QB_0)P + (\mathbf{V}BQ)(\Pi_+P)(\mathbf{V}QB_0)P + BP(\mathbf{V}QB_0)(P\Pi_-)(\mathbf{V}Q) + (B\mathbf{V}Q) \\ \times (\Pi_+P\Pi_-)(\mathbf{V}Q) = \begin{pmatrix} (e^{2V}-1)U_4 & 0 & 0 & (e^{2V}-1)U_3 \\ 0 & (e^{2V}-1)U_1 & (e^{2V}-1)U_2 & 0 \\ 0 & (e^{-2V}-1)U_3 & (e^{-2V}-1)U_4 & 0 \\ (e^{-2V}-1)U_2 & 0 & 0 & (e^{-2V}-1)U_1 \end{pmatrix}. \quad (56)$$

Note that after the substitution (44) the external lines correspond to $\bar{\theta}^2 \theta^a \psi_a$. The sums of subdiagrams presented in Fig. 3 are given by the following expressions:

$$U_1 = \frac{1}{16} \bar{\theta}^2 \psi^2 \left(-\theta^2 \frac{2D^2}{\partial^2 + M^2} + \theta^a \frac{i(\gamma^\mu)_a{}^b \bar{D}_b D^2 \partial_\mu}{(\partial^2 + M^2)^2} + \frac{M^2 \bar{D}^2 D^2}{2(\partial^2 + M^2)^3} \right) + \text{terms without } \bar{\theta}^2 \\ = -\frac{1}{16} \bar{\theta}^2 \psi^2 \left[(y^\mu)^*, \left[y_\mu^*, \frac{\bar{D}^2 D^2}{16(\partial^2 + M^2)} \right] \right] + \text{terms without } \bar{\theta}^2; \\ U_2 = \frac{1}{16} \bar{\theta}^2 \psi^2 \left(-\theta^2 \frac{8M}{\partial^2 + M^2} + \frac{4iM}{(\partial^2 + M^2)^2} \theta^a (\gamma^\mu)_a{}^b \bar{D}_b \partial_\mu + \frac{2M^3}{(\partial^2 + M^2)^3} \bar{D}^2 \right) \\ + \text{terms without } \bar{\theta}^2 = -\frac{1}{16} \bar{\theta}^2 \psi^2 \left[(y^\mu)^*, \left[y_\mu^*, \frac{M \bar{D}^2}{4(\partial^2 + M^2)} \right] \right] + \text{terms without } \bar{\theta}^2; \\ U_3 = \bar{\theta}^2 \psi^2 \frac{M^3}{8(\partial^2 + M^2)^3} D^2 = -\frac{1}{16} \bar{\theta}^2 \psi^2 \left[(y^\mu)^*, \left[y_\mu^*, \frac{M D^2}{4(\partial^2 + M^2)} \right] \right]; \\ U_4 = \frac{1}{16} \bar{\theta}^2 \psi^2 \left(-\frac{2}{\partial^2 + M^2} D^2 \theta^2 - \frac{i(\gamma^\mu)_a{}^b}{(\partial^2 + M^2)^2} D^2 \bar{D}^a \theta_b \partial_\mu + \frac{M^2}{2(\partial^2 + M^2)^3} D^2 \bar{D}^2 \right) \\ + \text{terms without } \bar{\theta}^2 = -\frac{1}{16} \bar{\theta}^2 \psi^2 \left[(y^\mu)^*, \left[y_\mu^*, \frac{D^2 \bar{D}^2}{16(\partial^2 + M^2)} \right] \right] + \text{terms without } \bar{\theta}^2. \quad (57)$$

(Note that formally writing the equalities we omit possible singular contributions in the massless case, which will be discussed later in details.) Thus, in the matrix form the result can be formally presented as

$$BP(\mathbf{V}QB_0)P(\mathbf{V}QB_0)P + (\mathbf{V}BQ)(\Pi_+P)(\mathbf{V}QB_0)P + BP(\mathbf{V}QB_0)(P\Pi_-)(\mathbf{V}Q) \\ + (B\mathbf{V}Q)(\Pi_+P\Pi_-)(\mathbf{V}Q) = -\frac{1}{16} \bar{\theta}^2 \psi^2 \left[(y^\mu)^*, [y_\mu^*, I_0] \right] + \text{terms without } \bar{\theta}^2. \quad (58)$$

Let us substitute this expression into Eq. (41). Then the terms without $\bar{\theta}^2$ vanish. Really, the non-trivial result is obtained only if a supergraph contains θ^4 , but the functional integration can produce only θ -s (and cannot produce $\bar{\theta}$ -s, see Eq. (47)). Therefore,

$$A_1 = -\frac{1}{8} \text{Tr} \left\langle \bar{\theta}^2 \psi^2 \star [(y^\mu)^*, [y_\mu^*, I_0]] \right\rangle. \quad (59)$$

The expression A_2 is given by the last diagram in Fig. 2. It contains two effective vertices. Each of these effective vertices can be presented as a sum of subdiagrams presented in Fig. 1. As we discussed above, after the substitution (44) they can be written in the form (51).

Substituting two these expressions into Eq. (42) and taking into account vanishing of the terms which do not contain $\bar{\theta}^2$, we obtain

$$A_2 = -\frac{1}{8}\text{Tr}\left\langle\bar{\theta}^2\psi^2[(y^\mu)^*, I_0] \star [y_\mu^*, I_0] \star\right\rangle. \quad (60)$$

After some transformations similar to the ones described in [49], the sum of the contributions A_0 , A_1 , and A_2 can be presented in the form

$$A_0 + A_1 + A_2 = -\frac{1}{8}\text{Tr}\left\langle\bar{\theta}^2\psi^2[(y^\mu)^*, [y_\mu^*, \ln(\star)]]\right\rangle - \text{singularities}. \quad (61)$$

Note that deriving this equation it is necessary to commute $\bar{\theta}^2$ with \star and I_0 . Such commutators are no more than linear in $\bar{\theta}$ and, therefore, disappear after integration over the anticommuting variables.

The traces of commutators in Eq. (61) evidently vanish, but the nontrivial result is obtained due to singularities, as we explain below. Note that traces of θ commutators do not produce the singularities, so that in Eq. (61) we can write x^μ instead of $(y^\mu)^*$. Therefore, the final result for the considered expression can be presented in the form

$$\begin{aligned} \frac{d\Delta\Gamma_{\mathbf{V}}^{(2)}}{d\ln\Lambda}\Big|_{\mathbf{V}=\bar{\theta}^2\theta^b\psi_b} &= \frac{i}{8}N_f^2\left\langle\left(\text{Tr}\sum_{I=0}^m c_I\bar{\theta}^{\dot{a}}(\gamma^\mu)_{\dot{a}}{}^b\psi_b\tilde{Q}[x_\mu, \ln(\star_I)]\right)^2\right\rangle \\ &- \frac{iN_f}{8}\frac{d}{d\ln\Lambda}\sum_{I=0}^m c_I\text{Tr}\left\langle\bar{\theta}^2\psi^2[x^\mu, [x_\mu, \ln(\star)]]\right\rangle_I - \text{singularities} + O(m_0^2). \end{aligned} \quad (62)$$

We see that in the momentum representation the right hand side is given by integrals of double total derivatives, because the commutator with x^μ corresponds to the derivative with respect to the momentum of the loop to which the external lines are attached. The trace gives integration over this momentum. However, the integrals of total derivatives do not vanish, because the integrands contain singularities. Really, if f is a function, which rapidly tends to 0 at infinity, and q^μ is the Euclidean momentum, then

$$\begin{aligned} \int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \left(\frac{q^\mu}{q^4} f(q^2) \right) &= 2\pi^2 \int \frac{dq^2}{(2\pi)^4} \frac{d}{dq^2} f(q^2) = -\frac{1}{8\pi^2} f(0) \\ &= -2\pi^2 \int \frac{d^4q}{(2\pi)^4} f(q^2) \delta^4(q) = - \int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \left(\frac{q^\mu}{q^4} \right) f(q^2). \end{aligned} \quad (63)$$

Note that in the left hand side of this equation, by definition, we can commute $\partial/\partial q^\mu$ and q^μ/q^4 , because a small vicinity of $q = 0$ is not included into the integration domain. Now, let us define the operator $\partial/\partial q^\mu$ which, by definition, does not commute with q^μ/q^4 and satisfies the relation

$$\int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} A^\mu = \text{Tr}[x_\mu, A^\mu] = 0. \quad (64)$$

Then Eq. (63) can be rewritten in the form

$$\int \frac{d^4q}{(2\pi)^4} \frac{q^\mu}{q^4} \frac{\partial}{\partial q^\mu} f(q^2) = \int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \left(\frac{q^\mu}{q^4} f(q^2) \right) - \int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \left(\frac{q^\mu}{q^4} \right) f(q^2). \quad (65)$$

The first term in the right hand side corresponds to the trace of the commutator in Eq. (62) and vanishes, and the second one comes from the singularity.

4 Exact expression for the photino mass RG function

In the previous section we obtained that the loop integrals which determine the renormalization of the photino mass are integrals of double total derivatives in the limit of the vanishing external momentum. However, these integrals do not vanish, because the integrands contain singularities proportional to $1/q^2$, where q is the Euclidean momentum. In this section we find the sum of singular contributions and compare it with the two-point function of the matter superfields.

First, we note that contributions of the massive Pauli–Villars superfields cannot contain singularities proportional to $1/q^2$. Therefore, only the terms corresponding to $I = 0$ (for which $c_0 = -1$ and $M_0 = 0$) in Eq. (62) do not vanish. Singularities cannot also appear in the singlet contribution, because there are only the first derivatives of $1/q^2$ in it. Therefore, singularities can arise only in the non-singlet contribution for $I = 0$. In this case

$$\star = \begin{pmatrix} \bar{*} & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & \tilde{*} & 0 \\ 0 & 0 & 0 & \tilde{*} \end{pmatrix}, \quad (66)$$

where

$$\begin{aligned} * &\equiv \frac{1}{1 - (e^{2V} - 1)\bar{D}^2 D^2 / 16\partial^2}; & \bar{*} &\equiv \frac{1}{1 - (e^{2V} - 1)D^2 \bar{D}^2 / 16\partial^2}; \\ \tilde{*} &\equiv \frac{1}{1 - (e^{-2V} - 1)\bar{D}^2 D^2 / 16\partial^2}; & \tilde{\bar{*}} &\equiv \frac{1}{1 - (e^{-2V} - 1)D^2 \bar{D}^2 / 16\partial^2}. \end{aligned} \quad (67)$$

By making the substitution $V \rightarrow -V$ in the generating functional, it is easy to see that the contributions containing $*$ and $\tilde{*}$ ($\bar{*}$ and $\tilde{\bar{*}}$) are equal. Moreover, it is possible to verify that the contributions of $*$ and $\bar{*}$ are also equal. To see this, one should note that they can be related by reversing the sequence of the operators D , \bar{D} etc. Therefore, we obtain

$$\left. \frac{d\Delta\Gamma_{\mathbf{V}}^{(2)}}{d\ln\Lambda} \right|_{\mathbf{V}=\bar{\theta}^2\theta^a\psi_a} = \frac{iN_f}{2} \frac{d}{d\ln\Lambda} \text{Tr} \psi^2 \left\langle \bar{\theta}^2 [(y_\mu)^*, [(y^\mu)^*, \ln(*)] \right\rangle - \text{singularities}. \quad (68)$$

The left hand side of this equation is related to the function $d_m^{-1}(\alpha_0, \Lambda/p)$ by Eq. (45). To calculate the expression in the right hand side, we start with calculating the inner commutator. Note that, due to the operator Tr , cyclic permutations in the expression which is commuted with the first $(y_\mu)^*$ do not change singular contributions. This follows from the fact that $\bar{\theta}^2$ in the above expression can be shifted to an arbitrary point of the considered supergraph (because the terms containing $\bar{\theta}$ in lower degrees evidently vanish.) Taking into account the possibility of making these cyclic permutations, we can write the result of calculating the inner commutator in the form

$$\begin{aligned} -\frac{1}{8\pi} \mathcal{V}_\psi \cdot \frac{d}{d\ln\Lambda} \left(\frac{m_0}{d_m(\alpha_0, \Lambda/p)} - \frac{m_0}{\alpha_0} \right) \Big|_{p=0} &= N_f \frac{d}{d\ln\Lambda} \text{Tr} \psi^2 \left\langle \bar{\theta}^2 \left[(y_\mu)^*, i(e^{2V} - 1) \frac{\bar{D}^2 D^2 \partial_\mu}{16\partial^4} * \right. \right. \\ &\quad \left. \left. - (\gamma^\mu)^{cd} (e^{2V} - 1) \theta_c \frac{\bar{D}_d D^2}{8\partial^2} * \right] \right\rangle - \text{singularities}. \end{aligned} \quad (69)$$

The trace of the commutator is evidently equal to 0, but the result does not vanish due to singularities, which can appear both from the first term and from the second term. To calculate the singularity of the first term, we use the identity

$$\left[(y^\mu)^*, \frac{\partial_\mu}{\partial^4}\right] = \left[-i\frac{\partial}{\partial q_\mu}, -\frac{iq_\mu}{q^4}\right] = -2\pi^2\delta^4(q_E) = -2\pi^2i\delta^4(q). \quad (70)$$

Then the contribution of the first term in Eq. (69) can be presented as

$$\begin{aligned} & \frac{\pi^2}{8}N_f \frac{d}{d\ln\Lambda} \text{Tr} \psi^2 \left\langle \bar{\theta}^2 * (e^{2V} - 1) \bar{D}^2 D^2 \delta^4(\partial) \right\rangle \\ &= -\frac{\pi^2}{128}N_f \frac{d}{d\ln\Lambda} \text{Tr} \psi^2 \left\langle \bar{\theta}^2 * (e^{2V} - 1) \frac{\bar{D}^2 D^2}{\partial^2} \delta^4(\partial) \bar{D}^2 D^2 \right\rangle. \end{aligned} \quad (71)$$

Terms proportional to the first degree of m_0 in this expression can be expressed via the two-point Green function of the matter superfields. To do this, we note that the operator Tr contains the integration over $d^4\theta$. The non-trivial result can be obtained only if the integrand contains $\theta^4 = \bar{\theta}^2 \cdot \theta^2$. Therefore, the covariant derivatives cannot act to $\bar{\theta}^2$ explicitly written in Eq. (71), and the expression (71) can be presented in the form

$$-\frac{\pi^2}{128}N_f \frac{d}{d\ln\Lambda} \text{Tr} \psi^2 \left\langle \bar{\theta}^2 \bar{D}^2 D^2 * (e^{2V} - 1) \frac{\bar{D}^2 D^2}{\partial^2} \delta^4(\partial) \right\rangle. \quad (72)$$

(As usual, we omit all terms with the derivatives of the slowly varying spinor ψ , which are suppressed by powers of Λ^{-1} .) In the momentum representation the argument of the δ -function becomes a loop momentum. In particular,

$$\delta^4(\partial)\delta^4(x-y) = \int \frac{d^4q}{(2\pi)^4} \delta^4(q) e^{-iq_\alpha(x^\alpha - y^\alpha)} = \frac{1}{(2\pi)^4}. \quad (73)$$

Using this identity we present the first singular contribution in Eq. (69) in the form

$$-\frac{N_f}{32\pi^2} \frac{d}{d\ln\Lambda} \int d^4x d^4y d^4\theta_x \psi_x^2 \bar{\theta}_x^2 \left\langle \frac{\bar{D}_x^2 D_x^2}{8} * \frac{\bar{D}_x^2 D_x^2}{8\partial^2} \delta_{xy}^8 \right\rangle \Big|_{\theta_y=\theta_x}. \quad (74)$$

From the other hand, differentiating Eq. (26) with respect to the sources j_x and j_y^* for each flavor we obtain

$$\left(\frac{\delta^2\Gamma}{\delta\phi_y^* \delta\phi_x}\right)^{-1} = -\frac{\delta^2 W}{\delta j_y^* \delta j_x} = \left\langle \frac{\bar{D}_x^2 D_x^2}{8\partial^2} * \frac{\bar{D}_x^2 D_x^2}{8\partial^2} \delta_{xy}^8 \right\rangle, \quad (75)$$

where (in the limit, when all masses except for m_0 vanish) the inverse Green function satisfies the condition

$$\int d^8y \frac{\delta^2\Gamma}{\delta\phi_x \delta\phi_y^*} \frac{\bar{D}_y^2}{8\partial^2} \left(\frac{\delta^2\Gamma}{\delta\phi_z \delta\phi_y^*}\right)^{-1} = -\frac{1}{2} \bar{D}_x^2 \delta_{xz}^8. \quad (76)$$

Therefore, it is possible to relate the considered expression to the two-point Green function of the matter superfields by the equation

$$-\frac{N_f}{32\pi^2} \frac{d}{d\ln\Lambda} \int d^4x d^4y d^4\theta_x \psi_x^2 \bar{\theta}_x^2 \partial_x^2 \left(\frac{\delta^2\Gamma}{\delta\phi_x \delta\phi_y^*}\right)^{-1} \Big|_{\theta_y=\theta_x}. \quad (77)$$

The two-point function of the matter superfields can be easily found by differentiating Eq. (20),

$$\frac{\delta^2\Gamma}{\delta\phi_x \delta\phi_y^*} = \frac{1}{16} D_y^2 \left(G + m_0 \theta^2 g + m_0 \bar{\theta}^2 g^* + m_0^2 \theta^4 \tilde{g} \right)_y \bar{D}_x^2 \delta_{xy}^8. \quad (78)$$

The corresponding inverse function can be found from Eq. (76). Taking into account that due to Eq. (47) the dependence on $\bar{\theta}$ disappears, the result can be written as

$$\left(\frac{\delta^2 \Gamma}{\delta \phi_x \delta \phi_y^*}\right)^{-1} \rightarrow -\frac{\bar{D}_x^2 D_y^2}{4\partial^2} (G + m_0 \theta^2 g)^{-1} \delta_{xy}^8. \quad (79)$$

Substituting the expression for the inverse Green function to Eq. (77) we obtain

$$\frac{N_f}{128\pi^2} \frac{d}{d \ln \Lambda} \int d^4 x d^4 y d^4 \theta_x \psi_x^2 \bar{\theta}_x^2 \bar{D}_x^2 D_y^2 (G + m_0 \theta^2 g)^{-1} \delta_{xy}^8 \Big|_{\theta_y = \theta_x}. \quad (80)$$

To simplify this expression, we note that the covariant derivatives do not act to the explicitly written θ -s, because the integral over $d^4 \theta$ does not vanish only if the integrand contains θ^4 . Therefore, they should act to δ_{xy}^8 ,

$$\bar{D}_x^2 D_y^2 \delta_{xy}^8 \Big|_{\theta_y = \theta_x} = 4\delta^4(x - y). \quad (81)$$

The coordinate δ -function allows calculating one of the coordinate integrals. The integrand in the remaining coordinate integral contains the spinor ψ , which slowly depends on the coordinates. In the momentum representation this implies that the corresponding momentum tends to 0. Therefore, using Eq. (46) we obtain that the first term of Eq. (69) gives

$$\mathcal{V}_\psi \cdot N_f \frac{1}{32\pi^2} \frac{d}{d \ln \Lambda} \int d^4 \theta \bar{\theta}^2 (G + m_0 \theta^2 g)^{-1} \Big|_{q=0} + O(m_0^2), \quad (82)$$

where the momentum q is an argument of the functions G and g . Note that this expression is not well-defined. The well-defined expression will be obtained after adding the contribution of the second term in Eq. (69).



Figure 4: Supergraphs which have coinciding momenta. The left supergraph corresponds to $n = 2$, and the right one schematically illustrates the $n = 6$ case. In the right diagram the circles denote the 1PI subdiagrams contributing to the anomalous dimension of the matter superfields.

Singularities coming from the second term appear only if a diagram has coinciding momenta in some propagators of the matter loop. An example of such a diagram is given in the left part of Fig. 4. From this figure, it is evident that coinciding momenta appear only if the corresponding graph can be made disconnected by two cuts of the matter line. By other words, it consists of some 1PI parts which are connected with each other only by the matter line, see the right part of Fig. 4, where these parts are denoted by the circles. Let us denote a number of these parts by n .

We have already mentioned that due to integration over $d^4 \theta$ the integrand should contain θ^4 . $\bar{\theta}^2$ is explicitly present in Eq. (69). θ^2 (or θ) is contained inside the gauge propagators (15) and vertices with smaller degrees of θ -s. In the first term of Eq. (69) θ^2 can appear from each of the n 1PI parts of the considered supergraph. In this term the covariant derivatives do not act to θ^2 , because then the degree of θ will be less than 4. Therefore, the expression for the first term contains

$$m_0 \theta^2 \left(g_1 \Delta G_2 \dots \Delta G_n + \Delta G_1 g_2 \dots \Delta G_n + \dots + \Delta G_1 \Delta G_2 \dots g_n \right), \quad (83)$$

where $G \equiv 1 + \Delta G$ and ΔG_i and g_i are contributions of 1PI parts to the functions G and g , respectively.

Analysing the second term of Eq. (69), first, we consider the case when the supersymmetric covariant derivatives do not act to θ^2 . In this case we obtain $(n - 1)$ contributions containing

$$-\frac{\bar{D}^2 D^2}{16\partial^2} \cdot (\gamma^\mu)^{cd} \theta_c \frac{\bar{D}_d D^2}{4\partial^2} = \frac{i\partial^\mu \bar{D}^2 D^2}{8\partial^4} \quad (84)$$

and 1 contribution proportional to $\theta^2 \theta_c = 0$. Therefore, (after symmetrization) the sum of the considered terms will be proportional to

$$-\frac{(n-1)}{n} m_0 \theta^2 \left(g_1 \Delta G_2 \dots \Delta G_n + \Delta G_1 g_2 \dots \Delta G_n + \dots + \Delta G_1 \Delta G_2 \dots g_n \right). \quad (85)$$

Also it is necessary to consider the case when the supersymmetric covariant derivatives act to θ^2 . Because the second term of Eq. (69) is linear in the explicitly written θ , only one D can act to θ^2 . However, from the dimensional and chirality considerations it is possible to see that in this case the supergraph contains the structures like

$$\theta_a \cdot D^2 \cdot (\gamma^\mu)^{cd} \theta_c \frac{\bar{D}_d D^2}{16\partial^2} = O(\theta) \quad \text{or} \quad (\gamma^\nu)^{ab} \theta_a \bar{D}_b \cdot (\gamma^\mu)^{cd} \theta_c \frac{\bar{D}_d D^2}{16\partial^2} = \theta^2 \frac{\bar{D}^2 D^2}{32\partial^2}, \quad (86)$$

where $O(\theta)$ denotes terms linear in θ and terms without θ . In the first case the corresponding contributions vanish, and there are no singularities (coming from ∂_μ/∂^4) in the second case.

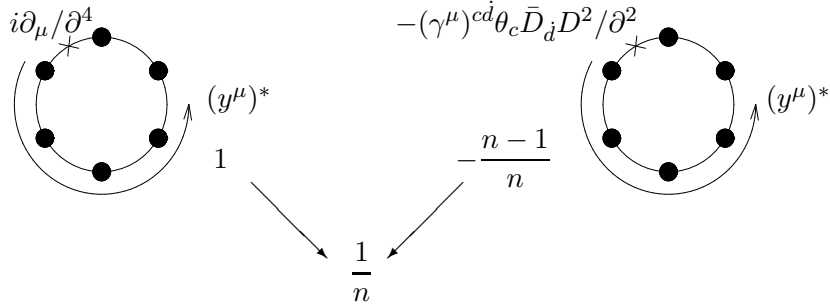


Figure 5: Summation of singularities.

Consequently, comparing Eqs. (83) and (85), we see that the contribution of the second term in Eq. (69) is equal to the contribution of the first term multiplied by $(1 - n)/n$. This implies that the sum of both terms is in

$$1 - \frac{n-1}{n} = \frac{1}{n} \quad (87)$$

times larger than the contribution of the first term. The expression for first term (given by Eq. (82)) contains

$$(G + m_0 \theta^2 g)^{-1} = (1 + \Delta G + m_0 \theta^2 g)^{-1} \rightarrow m_0 \theta^2 \sum_{n=1}^{\infty} n (-1)^n g (\Delta G)^{n-1}, \quad (88)$$

where the arrow points that we omit terms without θ^2 , which give vanishing contributions. Therefore, the sum of both terms is proportional to

$$m_0 \theta^2 \sum_{n=1}^{\infty} (-1)^n g (\Delta G)^{n-1} = -m_0 \theta^2 g (1 + \Delta G)^{-1} = -m_0 \theta^2 g G^{-1}. \quad (89)$$

Thus, in order to find the sum of all singular terms, it is necessary to replace $(G + m_0\theta^2 g)^{-1}$ in Eq. (82) by $-m_0\theta^2 G^{-1}g$. The integration over the anticommuting variable θ gives the multiplier 4. Therefore, if we denote the momentum by q , the all-orders result for the expression (45) takes the form

$$- \mathcal{V}_\psi \cdot \frac{N_f}{8\pi^2} \frac{d}{d \ln \Lambda} \left(m_0 G(\alpha_0, \Lambda/q)^{-1} g(\alpha_0, \Lambda/q) \right) \Big|_{q=0} + O(m_0^2). \quad (90)$$

Note that in Eq. (45) the differentiation with respect to $\ln \Lambda$ is made at fixed values of the renormalized coupling constant and the renormalized photino mass. The first term in the right hand side of Eq. (45) is a finite function of these parameters. This implies that its derivative with respect to $\ln \Lambda$ vanishes. Therefore, comparing Eqs. (45) and (90) in the limit $m_0 \rightarrow 0$, we obtain the equation which does not contain the auxiliary parameter $\mathcal{V}_\psi \rightarrow \infty$,

$$\frac{d}{d \ln \Lambda} \left(\frac{m_0}{\alpha_0} \right) = - \frac{N_f}{\pi} \frac{d}{d \ln \Lambda} \left(m_0 G(\alpha_0, \Lambda/q)^{-1} g(\alpha_0, \Lambda/q) \right) \Big|_{q=0}. \quad (91)$$

The dependence of supergraphs on m_0 and α_0 comes from the propagator of the gauge superfield (15) and the vertices with smaller degrees of θ -s. The function $g(\alpha_0, \Lambda/q)$ is determined by terms proportional to $m_0\theta^2$. Therefore, the vertices in the second string of Eq. (14) and terms proportional to m_0 without θ^2 in the gauge propagators (15) do not contribute to this function. Consequently, from Eq. (15) it is evident that the $m_0\theta^2$ terms can be obtained by differentiating the superdiagram for the rigid theory with respect to $\ln \alpha_0$, namely,

$$g(\alpha_0, \Lambda/q) = \alpha_0 \frac{\partial}{\partial \alpha_0} G(\alpha_0, \Lambda/q). \quad (92)$$

This implies that the right hand side of Eq. (91) can be presented in the form

$$\begin{aligned} - \frac{N_f}{\pi} \frac{d}{d \ln \Lambda} \left(m_0 \alpha_0 \frac{\partial}{\partial \alpha_0} \ln G(\alpha_0, \Lambda/q) \right) \Big|_{q=0} &= - \frac{N_f}{\pi} \frac{m_0}{\alpha_0} \frac{d}{d \ln \Lambda} \left(\alpha_0^2 \frac{\partial}{\partial \alpha_0} \ln G(\alpha_0, \Lambda/q) \right) \Big|_{q=0} \\ &- \frac{N_f}{\pi} \frac{d}{d \ln \Lambda} \left(\frac{m_0}{\alpha_0} \right) \alpha_0^2 \frac{\partial}{\partial \alpha_0} \ln G(\alpha_0, \Lambda/q) \Big|_{q=0}. \end{aligned} \quad (93)$$

In this expression the total derivative $d/d \ln \Lambda$ acts to both explicitly written $\ln \Lambda$ and $\ln \Lambda$ inside α_0 . It can be expressed in terms of the partial derivatives as

$$\frac{d}{d \ln \Lambda} = \beta(\alpha_0) \frac{\partial}{\partial \alpha_0} + \frac{\partial}{\partial \ln \Lambda}, \quad (94)$$

where the partial derivative $\partial/\partial \ln \Lambda$ acts only to the explicitly written $\ln \Lambda$. By the help of Eq. (94) one can commute the derivatives entering Eq. (93), taking into account that

$$\gamma(\alpha_0) = - \frac{d \ln Z}{d \ln \Lambda} = \frac{d \ln G(\alpha_0, \Lambda/q)}{d \ln \Lambda} \Big|_{q=0}; \quad \beta(\alpha_0) = \frac{d \alpha_0}{d \ln \Lambda}. \quad (95)$$

As the result, we rewrite the expression (93) in the form

$$\begin{aligned} \frac{d}{d \ln \Lambda} \left(\frac{m_0}{\alpha_0} \right) &= - \frac{N_f}{\pi} \left[m_0 \alpha_0 \frac{d \gamma(\alpha_0)}{d \alpha_0} - m_0 \alpha_0^3 \frac{d}{d \alpha_0} \left(\frac{\beta(\alpha_0)}{\alpha_0^2} \right) \frac{\partial}{\partial \alpha_0} \ln G(\alpha_0, \Lambda/q) \right. \\ &\left. + \frac{d}{d \ln \Lambda} \left(\frac{m_0}{\alpha_0} \right) \alpha_0^2 \frac{\partial}{\partial \alpha_0} \ln G(\alpha_0, \Lambda/q) \Big|_{q=0} \right]. \end{aligned} \quad (96)$$

With the higher derivative regularization the β -function and the anomalous dimension of the matter superfields defined in terms of the bare coupling constant satisfy the NSVZ relation in

all orders (in an arbitrary subtraction scheme). This was proved in [25, 26] by direct summation of supergraphs. Therefore, it is possible to use Eq. (5), so that

$$\frac{d}{d\alpha_0} \left(\frac{\beta(\alpha_0)}{\alpha_0^2} \right) = -\frac{N_f}{\pi} \frac{d\gamma(\alpha_0)}{d\alpha_0}. \quad (97)$$

We substitute this expression into Eq. (96). This gives

$$\begin{aligned} \frac{d}{d\ln\Lambda} \left(\frac{m_0}{\alpha_0} \right) = & -\frac{N_f}{\pi} \frac{d}{d\ln\Lambda} \left(\frac{m_0}{\alpha_0} \right) \alpha_0^2 \frac{\partial}{\partial\alpha_0} \ln G(\alpha_0, \Lambda/q) \Big|_{q=0} \\ & -\frac{N_f}{\pi} m_0 \alpha_0 \frac{d\gamma(\alpha_0)}{d\alpha_0} \left[1 + \frac{N_f}{\pi} \alpha_0^2 \frac{\partial}{\partial\alpha_0} \ln G(\alpha_0, \Lambda/q) \Big|_{q=0} \right]. \end{aligned} \quad (98)$$

Solving this equation for $d(m_0/\alpha_0)/d\ln\Lambda$, we obtain Eq. (6),

$$\frac{d}{d\ln\Lambda} \left(\frac{m_0}{\alpha_0} \right) = -\frac{m_0 \alpha_0 N_f}{\pi} \cdot \frac{d\gamma(\alpha_0)}{d\alpha_0}. \quad (99)$$

Following Ref. [5], it is possible to introduce the θ -dependent coupling constant

$$\frac{1}{\alpha_0} (1 - 2m_0\theta^2) = \frac{1}{\alpha_0(1 + 2m_0\theta^2)}. \quad (100)$$

Then the result for the photino mass renormalization can be combined with the NSVZ β -function in the equation

$$\frac{d}{d\ln\Lambda} \left(\frac{1}{(1 + 2m_0\theta^2)\alpha_0} \right) = -\frac{N_f}{\pi} \left[1 - \gamma((1 + 2m_0\theta^2)\alpha_0) \right], \quad (101)$$

which agrees with the general arguments based on the Statement presented in [7].

5 Explicit two-loop calculation

In this section we explicitly calculate the function $d(m_0/\alpha_0)/d\ln\Lambda$ in the two-loop approximation and demonstrate that it is given by integrals of double total derivatives. Moreover, we compare the two-loop result for this function with the one-loop result for the anomalous dimension of the matter superfields. For simplicity, in this section we will use the gauge

$$\frac{\xi_0}{K} = \frac{1}{R}. \quad (102)$$

Making the calculation we will omit all terms proportional to $(m_0)^n$ with $n \geq 2$, but keep the terms linear in m_0 . That is why it is convenient to consider terms linear in m_0 as vertexes.⁵ This implies that the propagator of the gauge superfield is the same as in the rigid theory, namely, it is proportional to

$$\frac{e_0^2}{\partial^2 R (\partial^2/\Lambda^2)} \delta_{xy}^8, \quad (103)$$

and the vertex linear in m_0 is given by

⁵Note that this method of calculation is different from the one used in the previous sections. Consequently, we will be able to verify independently the general arguments described above.

$$\begin{aligned} \frac{m_0}{32e_0^2} \int d^4x d^4\theta \Big(\theta^2 D^a V R(\partial^2/\Lambda^2) \bar{D}^2 D_a V + \bar{\theta}^2 \bar{D}^{\dot{a}} V R(\partial^2/\Lambda^2) D^2 \bar{D}_{\dot{a}} V \\ + \theta^2 V R(\partial^2/\Lambda^2) \bar{D}^2 D^2 V + \bar{\theta}^2 V R(\partial^2/\Lambda^2) D^2 \bar{D}^2 V \Big). \end{aligned} \quad (104)$$

We will graphically denote this vertex by a cross on a wavy line.

Let us start with the two-point Green function of the matter superfields. For the rigid theory, regularized by higher derivatives, in the one-loop approximation it was calculated, e.g. in [29], and can be written as

$$G(\alpha_0, \Lambda/p) = 1 - \int \frac{d^4k}{(2\pi)^4} \frac{2e_0^2}{R_k k^2 (k+p)^2} + O(e_0^4), \quad (105)$$

where $R_k \equiv R(k^2/\Lambda^2)$. Then the one-loop anomalous dimension (defined in terms of the bare coupling constant) is

$$\gamma(\alpha_0) = -\frac{d \ln Z}{d \ln \Lambda} = \frac{d \ln G}{d \ln \Lambda} \Big|_{p=0} = -e_0^2 \int \frac{d^4k}{(2\pi)^4} \frac{d}{d \ln \Lambda} \frac{2}{R_k k^4} + O(e_0^4) = -\frac{\alpha_0}{\pi} + O(\alpha_0^2). \quad (106)$$

The part of the matter superfield two-point function proportional to $(m_0)^1$ is determined by two diagrams presented in Fig. 6. Each of them contains a single vertex (104). We will not consider diagrams with larger numbers of such vertices, because their contributions are proportional to $(m_0)^n$ with $n \geq 2$. After calculating the diagrams presented in Fig. 6 we obtain the one-loop contribution to the function $g(\alpha_0, \Lambda/p)$, which is defined by Eq. (20). The result is written as



Figure 6: These diagrams give the θ -dependent part of the two-point Green function of the matter superfields in the lowest order in m_0 .

$$g(\alpha_0, \Lambda/p) = - \int \frac{d^4k}{(2\pi)^4} \frac{2e_0^2}{R_k k^2 (k+p)^2} + O(e_0^4). \quad (107)$$

Comparing Eqs. (105) and (107), it is easy to verify correctness of Eq. (92), which has essentially been used in deriving the exact expression for the photino mass renormalization. From Eqs. (105) and (107) we also conclude that the θ -dependent renormalization constant for the matter superfield can be chosen in the form

$$\mathcal{Z} = 1 + \frac{\alpha}{2\pi} (1 + 2m_0\theta^2) \ln \frac{\Lambda}{\mu} + \frac{\alpha}{2\pi} (b_1 + 2m_0\theta^2 \tilde{b}_1) + O(\alpha^2), \quad (108)$$

where b_1 and \tilde{b}_1 are finite constants, which fix a subtraction scheme in the considered approximation.

Now, let us verify Eq. (62). The singlet contribution is evidently absent in the considered approximation, so that the first term vanishes. For calculating the second term, we consider the expression

$$-\frac{i}{8} N_f \frac{d}{d \ln \Lambda} \sum_{I=0}^m c_I \text{tr} \langle \bar{\theta}^2 \psi^2 \ln(\star) \rangle_I, \quad (109)$$

where tr denotes the usual matrix trace. (Unlike Tr , it does not include integration over the superspace.) To calculate it in the considered approximation, we use Eq. (28),

$$\ln(\star) = -\ln(1 - I_0) = I_0 + \frac{1}{2}(I_0)^2 + \dots = BP + \frac{1}{2}BPBP + \dots \quad (110)$$



Figure 7: The supergraphs contributing to the $\text{tr}\langle \ln(\star) \rangle$ in the considered approximation.

The vertices B contain the gauge superfield V . The functional integration encoded in the angular brackets transforms V -s into the gauge propagators. The matrices P give propagators of the matter superfields. Then, it is easy to see that from Eq. (110) in the lowest approximation we obtain the supergraphs presented in Fig. 7. (Note that we omit all diagrams proportional to $(m_0)^n$ with $n \geq 2$.) They are similar to the diagrams which contribute to the anomalous dimension of the matter superfields. Having calculated them we obtained (after the Wick rotation in the Euclidean space)

$$\begin{aligned} \text{tr}\langle \bar{\theta}^2 \psi^2 \ln(\star) \rangle &= -\bar{\theta}_x^2 \psi_x^2 \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{(1 + m_0 \theta^2) e_0^2}{k^2 R_k (q^2 + M^2) ((q + k)^2 + M^2)} \\ &\times \frac{i}{8} (\bar{D}^2 D^2 + D^2 \bar{D}^2) e^{-iq_\alpha (x^\alpha - y^\alpha)} \delta^4(\theta_x - \theta_y). \end{aligned} \quad (111)$$

In the momentum representation $[x^\mu, \dots]$ is written as $-i\partial/\partial q_\mu$. Therefore, after the Wick rotation in the Euclidean space

$$[x^\mu, [x_\mu, Y]] \rightarrow \frac{\partial}{\partial q_\mu} \frac{\partial}{\partial q^\mu} Y. \quad (112)$$

Using this equation and calculating the remaining integrals in Eq. (62), we can predict the result for the renormalization of the photino mass in the two-loop approximation. Taking into account that the singlet contribution vanishes, we expect that it is described by the function

$$\begin{aligned} \frac{d}{d \ln \Lambda} \left(\frac{m_0}{d_m(\alpha_0, \Lambda/p)} - \frac{m_0}{\alpha_0} \right) \Big|_{p=0; \alpha, m=\text{const}} &= 16\pi^2 m \alpha N_f \sum_{I=0}^m c_I \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \\ &\times \frac{d}{d \ln \Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \frac{1}{R_k k^2 (q^2 + M_I^2) ((k + q)^2 + M_I^2)} + O(\alpha^2). \end{aligned} \quad (113)$$

The function d_m^{-1} defined by Eq. (19) in the two-loop approximation is given by the sum of the diagrams presented in Fig. 8. Note that we calculate only terms proportional to first power of m_0 . The masses inside the function d_m^{-1} (certainly, except for the masses of the Pauli–Villars fields) are set to 0. The expressions which were obtained after calculating the diagrams presented in Fig. 8 are given in Appendix A. These expressions contain 4 different structures, only one surviving in their sum in agreement with the Ward identity. (Cancellation of the terms which do not satisfy the Ward identity can be considered as a non-trivial test of the calculation.)

The sum of the diagrams presented in Fig. 8 is given by Eq. (126). Using this equation one can find the function d_m^{-1} in the considered approximation. After the Wick rotation it can be written as

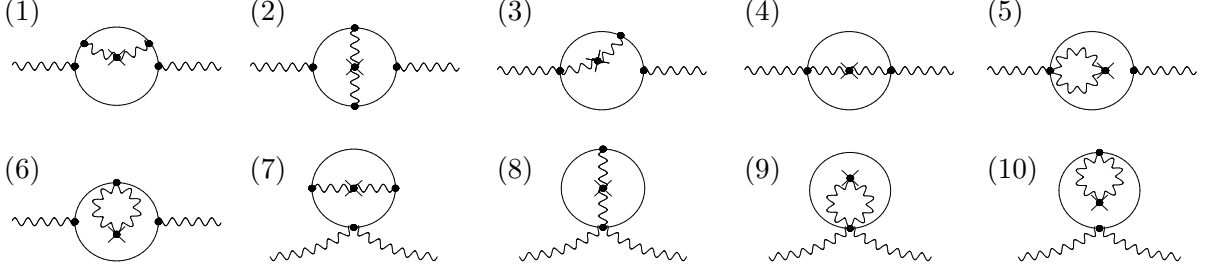


Figure 8: These diagrams define renormalization of the photino mass in the two-loop approximation in the limit $p \gg m_0$.

$$\begin{aligned}
d_m^{-1}(\alpha_0, \Lambda/p) &= \alpha_0^{-1} + 64\pi^2 \alpha_0 N_f \sum_{I=0}^m c_I \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{R_k k^2} \left(\frac{2q^\mu (q+k)_\mu - p^2}{(q^2 + M_I^2) ((q+p)^2 + M_I^2)} \right. \\
&\quad \times \frac{1}{((q+k)^2 + M_I^2) ((q+k+p)^2 + M_I^2)} - \frac{4M_I^2}{(q^2 + M_I^2)^2 ((q+p)^2 + M_I^2) ((q+k)^2 + M_I^2)} \Bigg) \\
&\quad + O(\alpha_0^2),
\end{aligned} \tag{114}$$

where p and k are the Euclidean momenta. To calculate the RG function $d(m_0/\alpha_0)/d \ln \Lambda$, we consider the expression (43). In the considered approximation it is written as

$$\begin{aligned}
\frac{d}{d \ln \Lambda} \left(\frac{m_0}{d_m(\alpha_0, \Lambda/p)} - \frac{m_0}{\alpha_0} \right) \Big|_{p=0; \alpha, m=\text{const}} &= 16\pi^2 m \alpha N_f \sum_{I=0}^m c_I \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \\
&\times \frac{d}{d \ln \Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \frac{1}{R_k k^2 (q^2 + M_I^2) ((k+q)^2 + M_I^2)} + O(\alpha^2).
\end{aligned} \tag{115}$$

Therefore, by the explicit calculation we have demonstrated that the integrals defining the function (6) are integrals of double total derivatives in the momentum space. Comparing the result with Eq. (113) we verify that Eq. (62) is valid in the considered approximation. Thus, the explicit calculation confirms the general argumentation presented in this paper. The integral in Eq. (115) is well-defined due to the derivative with respect to $\ln \Lambda$, which makes this integral convergent in the infrared region. It can be calculated using the equation

$$\int \frac{d^4 q}{(2\pi)^4} \frac{q^\mu}{q^4} \frac{\partial}{\partial q^\mu} f(q^2) = -\frac{1}{8\pi^2} f(0), \tag{116}$$

where $f(q^2)$ is a function with a sufficiently rapid fall-off at infinity. (In the considered case this is ensured by the presence of R_k in the denominator.) Using this equation it is easy to see that all terms with $I \neq 0$ (which correspond to the diagrams with the Pauli-Villars loops) give vanishing contributions. Therefore, only the term with $I = 0$ (for which $c_0 = -1$, $M_0 = 0$) survives:

$$\begin{aligned}
\frac{d}{d \ln \Lambda} \left(\frac{m_0}{\alpha_0} \right) &= -64\pi^2 m \alpha N_f \int \frac{d^4 q}{(2\pi)^4} \frac{q^\mu}{q^4} \frac{\partial}{\partial q^\mu} \frac{d}{d \ln \Lambda} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{R_k k^2 (k+q)^2} + O(\alpha^2) \\
&= 8m_0 \alpha_0 N_f \frac{d}{d \ln \Lambda} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{R_k k^4} + O(\alpha_0^2),
\end{aligned} \tag{117}$$

where we took into account that the differentiation with respect to $\ln \Lambda$ (acting on R_k) should be made at fixed values of the renormalized coupling constant α and the renormalized photino mass m . Comparing this expression with the one-loop anomalous dimension of the matter superfields in the rigid theory calculated with the considered regularization, which is given by Eq. (106), we obtain

$$\frac{d}{d \ln \Lambda} \left(\frac{m_0}{\alpha_0} \right) = -\frac{m_0 \alpha_0 N_f}{\pi} \cdot \frac{d\gamma(\alpha_0)}{d\alpha_0} + O(\alpha_0^2) = \frac{m_0 \alpha_0 N_f}{\pi^2} + O(\alpha_0^2). \quad (118)$$

Therefore, by the explicit calculation we have verified that Eq. (6) is really valid in the two-loop approximation due to factorization of the corresponding integrals into integrals of double total derivatives. Moreover, we have demonstrated that the exact equation (62) is also valid in the considered approximation. Certainly, in the considered approximation the relation (4) is scheme-independent (or, similarly, Eq. (6) is regularization-independent). That is why it would be more interesting to consider the next order of the perturbation theory. However, the argumentation of this paper is similar to the one presented in [25] for the rigid theory, which was checked by the explicit three-loop calculation. Thus, we believe that in the next approximation no principal differences appear.

6 Conclusion

In this paper we have derived the exact expression for the anomalous dimension of the photino mass in softly broken $\mathcal{N} = 1$ SQED with N_f flavors by direct summation of supergraphs. The result, which was first proposed in [5], is obtained for the RG function defined in terms of the bare coupling constant in the case of using the higher derivative regularization independently of the subtraction scheme. It follows from the fact that all integrals which determine the renormalization of the photino mass are integrals of double total derivatives in the momentum space. This statement is proved in all orders. The integrals of double total derivatives do not vanish due to singularities of the integrands, which can be summed exactly. This sum gives the expression in the right hand side of Eq. (6).

The general results obtained in this paper have been verified by the explicit two-loop calculation. This calculation demonstrates that the integrals defining renormalization of the photino mass are really integrals of double total derivatives and coincide with the prediction of the exact Eq. (62).

The RG functions defined in terms of the renormalized coupling constant depend on the subtraction scheme, and, therefore, the relation (4) written in terms of the renormalized coupling constant is valid only in a special subtraction scheme. We believe that, for the theory regularized by higher derivatives, this scheme can be constructed on the base of Eq. (6) similarly to the case of the rigid theory considered in [13]. This work is now in progress.

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A Appendix

Here we present explicit expressions for the diagrams presented in Fig. 8:

$$(5) = (6) = (9) = (10) = 0; \quad (119)$$

$$(1) = m_0 e_0^2 N_f \sum_{I=0}^m c_I \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \int d^4 \theta \theta^2 \frac{1}{R_k k^2 ((q+p)^2 - M_I^2) ((q+k)^2 - M_I^2)} \\ \times \frac{1}{(q^2 - M_I^2)} \left\{ \frac{1}{4} \bar{D}^2 \mathbf{V}(p, \theta) D^2 \mathbf{V}(-p, \theta) - \frac{M_I^2}{2(q^2 - M_I^2)} D^a \mathbf{V}(p, \theta) \bar{D}^2 D_a \mathbf{V}(-p, \theta) \right. \\ \left. + 2(q+p)_\mu (\gamma^\mu)^{ab} \bar{D}_b \mathbf{V}(p, \theta) D_a \mathbf{V}(-p, \theta) + \mathbf{V}(p, \theta) \mathbf{V}(-p, \theta) \frac{4q^2}{(q^2 - M_I^2)} ((q+p)^2 - M_I^2) \right\} \\ + \text{c.c.}; \quad (120)$$

$$(2) = m_0 e_0^2 N_f \sum_{I=0}^m c_I \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \int d^4 \theta \theta^2 \frac{1}{R_k k^4 (q^2 - M_I^2) ((q+p)^2 - M_I^2)} \\ \times \left\{ \frac{1}{4} \bar{D}^2 \mathbf{V}(p, \theta) D^2 \mathbf{V}(-p, \theta) + \frac{1}{4} D^a \mathbf{V}(p, \theta) \bar{D}^2 D_a \mathbf{V}(-p, \theta) \left(1 - \frac{2k^2}{((q+k)^2 - M_I^2)} \right) \right. \\ \left. + \frac{k^2(k^2 + p^2 - 2M_I^2)}{2((q+k)^2 - M_I^2)((q+k+p)^2 - M_I^2)} \right) + \mathbf{V}(p, \theta) \mathbf{V}(-p, \theta) \frac{2(k-p)^2(q^2 - M_I^2)}{((q+k)^2 - M_I^2)} \\ - 2(\gamma^\mu)^{ab} \bar{D}_b \mathbf{V}(p, \theta) D_a \mathbf{V}(-p, \theta) \left(\frac{q_\mu k^\alpha (k-p)_\alpha - p_\mu q^\alpha (q+k)_\alpha + k_\mu q^\alpha (q+p)_\alpha}{((q+k)^2 - M_I^2)} \right. \\ \left. + \frac{M_I^2 (k+p)_\mu}{((q+k+p)^2 - M_I^2)} \right) \Big\} + \text{c.c.}; \quad (121)$$

$$(3) = m_0 e_0^2 N_f \sum_{I=0}^m c_I \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \int d^4 \theta \theta^2 \frac{1}{R_k k^4 ((q+p)^2 - M_I^2) ((q+k)^2 - M_I^2)} \\ \frac{1}{(q^2 - M_I^2)} \left\{ -\frac{1}{4} \bar{D}^2 \mathbf{V}(p, \theta) D^2 \mathbf{V}(-p, \theta) (q^2 + (q+k)^2 + k^2 - 2M_I^2) - \frac{1}{4} D^a \mathbf{V}(p, \theta) \right. \\ \times \bar{D}^2 D_a \mathbf{V}(-p, \theta) (q^2 + (q+k)^2 - k^2 - 2M_I^2) + 2(\gamma^\mu)^{ab} \bar{D}_b \mathbf{V}(p, \theta) D_a \mathbf{V}(-p, \theta) \left(-q_\mu k_\alpha p^\alpha \right. \\ \left. - p_\mu (2q^2 + q_\alpha k^\alpha + k^2 - 2M_I^2) + k_\mu (2q^2 + q^\alpha p_\alpha - 2M_I^2) \right) - 4\mathbf{V}(p, \theta) \mathbf{V}(-p, \theta) ((k-p)^2 \\ \times (q^2 - M_I^2) + k^2((q+p)^2 - M_I^2)) \Big\} + \text{c.c.}; \quad (122)$$

$$(4) = m_0 e_0^2 N_f \sum_{I=0}^m c_I \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \int d^4 \theta \theta^2 \frac{1}{R_k k^4 (q^2 - M_I^2) ((q+k+p)^2 - M_I^2)} \\ \times \left\{ \frac{1}{4} \bar{D}^2 \mathbf{V}(p, \theta) D^2 \mathbf{V}(-p, \theta) + \frac{1}{4} D^a \mathbf{V}(p, \theta) \bar{D}^2 D_a \mathbf{V}(-p, \theta) + 2(k+p)_\mu (\gamma^\mu)^{ab} \bar{D}_b \mathbf{V}(p, \theta) \right. \\ \left. \times D_a \mathbf{V}(-p, \theta) + 2\mathbf{V}(p, \theta) \mathbf{V}(-p, \theta) (k+p)^2 \right\} + \text{c.c.}; \quad (123)$$

$$(7) = -m_0 e_0^2 N_f \sum_{I=0}^m c_I \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \int d^4 \theta \theta^2$$

$$\times \mathbf{V}(p, \theta) \mathbf{V}(-p, \theta) \frac{2(q^2 + M_I^2)}{R_k k^2 (q^2 - M_I^2)^2 ((q+k)^2 - M_I^2)} + \text{c.c.}; \quad (124)$$

$$(8) = m_0 e_0^2 N_f \sum_{I=0}^m c_I \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \int d^4 \theta \theta^2 \\ \times \mathbf{V}(p, \theta) \mathbf{V}(-p, \theta) \frac{2}{R_k k^2 (q^2 - M_I^2) ((q+k)^2 - M_I^2)} + \text{c.c.} \quad (125)$$

We see that these expressions contain 4 different V -structures. However, the only combination $D^a \mathbf{V} \bar{D}^2 D_a \mathbf{V}$ is transversal. We have verified that for the other 3 structures the contributions coming from various diagrams cancel each other. Then, after some transformations, the result (for the contribution of the considered diagrams into the effective action) can be written as

$$-N_f \sum_{I=0}^m c_I \int \frac{d^4 p}{(2\pi)^4} d^4 \theta \left(m_0 \theta^2 D^a \mathbf{V}(p, \theta) \bar{D}^2 D_a \mathbf{V}(-p, \theta) + \text{c.c.} \right) \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \\ \times \frac{e_0^2}{8 R_k k^2} \left(\frac{2 q^\mu (q+k)_\mu - p^2}{(q^2 - M_I^2) ((q+p)^2 - M_I^2) ((q+k)^2 - M_I^2) ((q+k+p)^2 - M_I^2)} \right. \\ \left. + \frac{4 M_I^2}{(q^2 - M_I^2)^2 ((q+p)^2 - M_I^2) ((q+k)^2 - M_I^2)} \right). \quad (126)$$

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